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Determination of the charge of relativistic heavy primaries in cosmic radiation by δ-ray counting and a comparison with photometric identification

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With 8 figures in the text

SUMMARY

A charge determination of relativistic heavy primaries in the cosmic radiation has been undertaken by determining the δ-ray density, N_δ, of tracks in nuclear emulsions. The investigation was restricted to particles with well identified charges in the interval $2 \leqslant Z \leqslant 10$, which made it possible to examine the applicability of δ-ray counting in this region. The standard error in N_δ was found to be greater than expected from pure statistical fluctuations, the coefficient of variation introduced by other sources of error than statistical ones being 7 %. Possible sources of error, which may explain this increase are discussed as well as different psychological factors, which affect the reproducibility. The relation between N_δ and Z^2 is found to be linear. The results obtained from the δ-ray counting are compared with photometric measurements and with other charge spectra based on δ-ray counting.

Introduction

A photometric method of measuring the grain density of tracks in nuclear emulsions has been developed at Lund [1, 2]. During the application of this method to relativistic heavy primaries in cosmic radiation [3] the necessity arose of controlling the charge calibration by an independent method. This control was performed by determining the δ-ray density of most of the tracks used in the photometric investigation. The δ-ray counting was undertaken in such a way that a comparison between the δ-ray method and the photometric method would be feasible after providing the appropriate charge calibration. It turned out that the calibration assumed in the photometric investigation was correct, and that the resolution between adjacent charges in the charge interval $2 \leqslant Z \leqslant 14$ was complete. This complete resolution made it possible to examine for the first time the method of δ-ray counting on a well identified material of multiply charged particles. In this paper the results of the investigation will be presented together with a comparison of the two methods of charge determination.

The δ-ray method

In most investigations of the heavy primaries in cosmic radiation, undertaken in nuclear emulsions, the δ-ray density has been used for determining the charge of the cosmic ray particles.

The number of δ-rays, dn, per unit length and with δ-ray energies in the interval W, $W + dW$ is given by the transcribed Mott's formula [4]:

$$dn = \frac{2\pi N Z^2 e^4}{m c^2 \beta^2} \cdot \frac{dW}{W^2} \left[1 - \frac{1-\beta^2}{2} \cdot \frac{W}{m c^2} + \frac{\pi Z \beta}{137} \left(\frac{1-\beta^2}{2\beta^2} \cdot \frac{W}{m c^2}\right)^{\frac{1}{2}} \left(1 - \frac{1-\beta^2}{2\beta^2} \cdot \frac{W}{m c^2}\right)\right]$$

$\beta = \frac{v}{c}$

m = the electron mass

N = the electron density of the detector

e = the electronic charge

v = the particle velocity

c = the velocity of light.

For the electron energies used in practice the relativistic correction term in the square brackets may be neglected, in which case the integration between the energy limits W_1 and W_2 gives

$$n = \frac{2\pi N Z^2}{\beta^2} \left(\frac{e^2}{m c^2}\right)^2 \left(\frac{m c^2}{W_1} - \frac{m c^2}{W_2}\right).$$

The upper limit W_2 is set either by the sensitivity of the emulsion or by the difficulty to see tracks of low-ionizing electrons. The lower limit W_1 is set by the criterion used in δ-ray counting. The fact is that a δ-ray must have a certain minimum length in order to be acceptable. Two criteria have been used, the grain criterion stipulating that an accepted δ-ray must have at least 4 (sometimes 3) grains [4], and the range criterion stipulating that the δ-ray must have a certain extension ($\simeq$ 2 microns) from the central line of the track [5]. Both criteria correspond to $W_1 \simeq 10$ keV [6, 7]. Comparative studies on the reproducibility in the counting have been carried out by, among others, Voyvodic [6], who prefers the grain criterion, by Tidman *et al.* [7], who find that the range criterion is more reliable than the grain criterion, and by Norlind [8], who is of the opinion that both criteria give about equivalent results. Recommendations as to how to apply the criteria have been given by Tidman *et al.* [7], and by Young *et al.* [9].

Provided that the conventions in δ-ray counting are kept independent of the charge Z and velocity β, the energy limits W_1 and W_2 are constants. Thus the equation for the δ-ray density is reduced to

$$n = \text{const.} \frac{Z^2}{\beta^2}.$$

From the above equation it is evident that at relativistic velocities the δ-ray density is a single-valued function of the charge of the particle.

Experimental details

The investigation has been carried out in a stack of 40 stripped emulsions (Ilford G 5, thickness 600 microns), exposed in January 1955 over Texas, U.S.A., at a geomagnetic latitude of 41° N. The floating altitude of the balloon was 29 km and the duration of the flight was 8 hours. The blob density was found to be 19 blobs/100 microns for relativistic singly-charged particles. The background of slow electrons was fairly high. Further information about the exposure, processing, and scanning is given in [3] together with a report of charge determination according to the photometric method.

A binocular microscope with a turnable stage for adjusting the track along the x-axis of the stage has been used. The optics were of Leitz' manufacture, viz., an objective Ks 100×, and an eye-piece Periplan Mess 10×. The condenser lamp was provided with a green filter in order to diminish the strain on the eyes, and the illumination was adjusted to give good working conditions. According to O'Brien and Noon [10] the light intensity does not appreciably affect the results in δ-ray counting, especially if the range criterion is used.

The investigation was restricted to particles in the charge interval $2 \leqslant Z \leqslant 10$. The tracks were chosen at random from a sample of tracks identified by photometric measurements [3]. They were selected by a person who did not take part in the δ-ray counting himself. The counter did not know anything in advance about the charge.

The measurements

Some qualitative comparisons between the two counting criteria were undertaken at the beginning of the work. The grain criterion was found to be applicable to tracks of particles with low charges, but the uncertainty increased in the case of charges $Z \gtrsim 6$, where the central core of the track is almost completely filled and where, on account of this, it begins to be difficult to decide whether a grain belongs to the central core or to the δ-ray. By using the range criterion such difficulties do not occur. In consequence this criterion was chosen.

A δ-ray to be accepted had to fulfil two conditions:

(*a*) The projected length of the δ-ray on the horizontal plane through the track had to be larger than 2 scale-divisions on the eye-piece scale (1.3 microns), measured from the longitudinal axis of the track.

(*b*) The δ-ray should not have any gaps, e.g., only such δ-rays were counted which form a continuous grain row from the track core to the point determined by the condition (*a*).

The application of the condition (*b*) results in the upper energy limit W_2 being given a comparatively small value. Thus the δ-rays with great energy, which are often difficult to detect, are excluded and all the accepted electron tracks lie in a rather narrow energy interval. In this way the counting conventions are simplified at the same time as the counting speed is increased. The only unfavorable effect of this choice of energy interval is that the constant in the equation $n = \text{const. } Z^2/\beta^2$ is diminished. The resulting decrease of δ-ray density can however be compensated by an appropriate reduction of the lower energy limit W_1. This is done by decreasing the length of accepted δ-rays as stated in condition (*a*). The condition (*a*) is practicable only in conjunction with the condition (*b*), as the difficulties in deciding whether, for instance, three grains constitute a δ-ray or not are greatly reduced if these grains have to form a blob.

In addition to the two conditions which an accepted δ-ray had to fulfil the following counting conventions were used:

(*c*) In case of complicated blobs having more than one protrusion, each of these protrusions was counted as one δ-ray.

(*d*) A δ-ray crossing the central core of the track was counted as one only if it was clear from the grain configuration that the grains belong to the same δ-ray.

In determining the δ-ray density, the cell lengths for 100 δ-rays were registered. Thus the measurements were not made with constant cell length. This precaution was taken in order to ensure that the counter would not be influenced by the remaining length of the cell. Thus the counter was prevented from unconsciously adjusting the number of δ-rays counted in this length so that the resulting δ-ray density in the cell would correspond to the charge of the particle causing the track, as estimated by him visually.

In each plate only the central layer, corresponding at most to $\frac{2}{3}$ of the thickness of the pellicle, was used. As only tracks with track lengths $\geqslant 3$ mm per pellicle were accepted it has not been necessary in this investigation to correct the obtained δ-ray densities for the dip of the tracks [10].

All the counting was performed by one person throughout the whole investigation. Hence variations in measurement techniques from observer to observer were eliminated.

During the first part of the investigation, when the counting conventions were not considered to be sufficiently deeply rooted, every track was counted twice. The results were accepted only if the two measurements differed by less than 5 %. In the contrary case, new measurements were undertaken until consistent results were obtained. When the counter had aquired greater skill in the counting procedure, only one measurement was made per track.

Influence of psychological factors on the reproducibility

In the first part of the investigation, when every track was counted twice, each day's work was started with a reference track. This was moreover returned to during the day's work if the conventions had apparently glided, as seen by the fact that the two measurements on the same track did not agree. However, it soon turned out to be necessary to disregard the method involving fixed reference tracks, since the counter recognized these tracks and recollected the doubtful grain configurations which he earlier had counted as δ-rays. In such a way the reproducibility of the method could easily have been overestimated. In order to avoid this effect each day's work was started with a track which was new to the counter. The charge of the particle causing the track (as a rule $Z = 6$) was announced to him. The counting conventions were considered to be correct only if the δ-ray density of the track agreed within the limits of statistical fluctuations with the mean δ-ray density of this charge.

Sudden daily changes in the δ-ray density were noted, showing marked correlations with the mental condition of the counter. Normal results were obtained only if the counter felt collected and relaxed without being affected by private anxieties or irritated by his surroundings. As an instance of this an interesting case may be quoted. After having attained normal results the counter was subjected to disturbance in the form of a conversation in his vicinity while the counting was in progress. Four particles in the charge interval $6 \leqslant Z \leqslant 8$ were δ-ray counted repeatedly in the period. It turned out that in all cases the charge was identified about one unit too high. Thus, in conventional work misidentifications would have been inevitable, if no reference tracks had been measured during the disturbance. It seems to be very remarkable that the new counting level was constant. The mean spread in δ-ray density during the period of disturbance was found to be $(1.5 \pm 1.3)\,\%$. The δ-ray densities determined under circumstances when the conventions obviously had glided, were excluded in the treatment of the material.

In the very beginning of the measurements it turned out that the results in δ-ray counting were most consistent if the number of tracks presented to the counter at one time did not exceed the amount which he was capable of counting in a day's work. In the contrary case, results were impaired, possibly owing to more careless counting taking place in order to get all the material finished; in addition, the counter may have felt that he was not really up to his task. This feeling could increase the uncertainty through autosuggestion. Similar effects are probably not met with in ordinary δ-ray counting, as the act of presenting to the counter a number of tracks, the charge of which is known to all but the counter, as done in our case, may be interpreted by the counter as some kind of personal test. Consequently, he is working under a greater psychological stress than is usual in ordinary δ-ray counting, and to a certain extent this must affect the results.

The measurements were made in two periods with a pause of about $2\frac{1}{2}$ months. It turned out that the counter could not adapt his conventions to earlier results after the pause. An analysis of all the δ-ray densities shows that the change in the counting level is positive and amounts to $(12 \pm 5)\,\%$. The material has been corrected for this effect.

The dependence of the δ-ray density on the charge

The cut-off energy at a geomagnetic latitude of $\lambda = 41°$ is 1.5 GeV/nucleon, corresponding to $\beta = 0.93$, and therefore the equation

$$n = \text{const.}\, Z^2$$

must be valid for the δ-ray density with good approximation. In Figure 1 the δ-ray density is plotted against the square of the charge assumed in the photometric investigation [3]. The total number of δ-rays counted in each charge group varies from 100 for $Z = 2$ to 4600 for $Z = 10$, thus ensuring that the statistical errors in each group are practically the same. The line drawn in the figure represents the best fit according to the method of least squares, assuming that all the charge groups are given the same statistical weight. As is seen in the figure, the experimental points fall very close to the straight line. Thus it can be assumed that the charge calibration adopted in the photometric investigation is probably right.

Re-arranging of the points to other charge values with preserved linearity between the points is, however, possible in one additional way, viz., by assuming that only the points He and Li are correctly identified, that no tracks of Be particles are found and that the remaining points in the figure belong to consecutive charges. The last condition is well established by the photometric investigation [3]. In this way a fairly good linearity is found as well, only the charges $Z = 6$ and 11 (in the new scale)

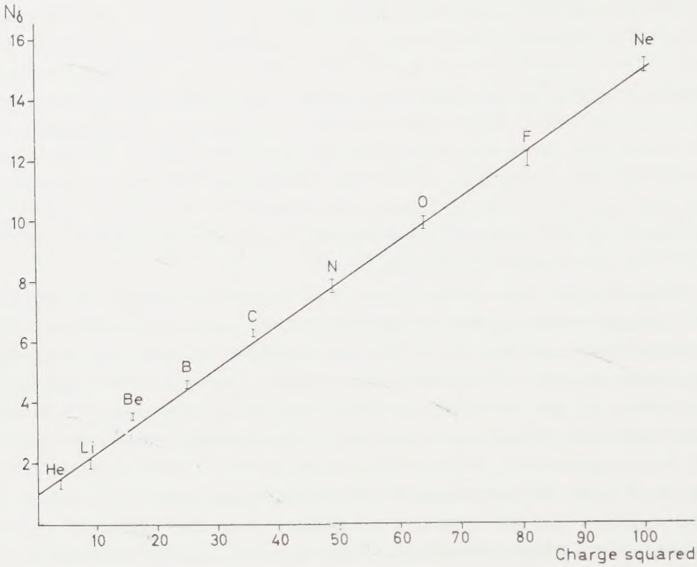

Fig. 1. The δ-ray density N_δ (δ-rays per 100 microns) versus the square of the charge assumed in the photometric investigation.

lying slightly off the new calibration curve. Nevertheless, there are several striking facts that speak against this attempted calibration:

(*a*) In the charge interval $4 \leqslant Z \leqslant 10$ we have found four interactions in which only fragments with charge $Z \leqslant 3$ are produced and in which charge balance is established between these relativistic fragments and the splitting nucleus if the charge calibration given in Figure 1 is used. These fragmentations are shown in Table 1. The attempted calibration curve would cause the charge of the primaries to lie one unit higher than indicated in Table 1. As the values shown in the table are upper limits of the charge, this attempted calibration curve cannot be correct.

Table 1. Charge indicating interactions in the charge interval $4 \leqslant Z \leqslant 10$.

Charge of the primary particle as determined in the photometric investigation	Assumed interaction
6	$C \rightarrow 2\alpha + 2\,p$
6	$C \rightarrow 2\alpha + 2\,p$
6	$C \rightarrow 1\,Li + 3\,p$
10	$Ne \rightarrow 3\alpha + 4\,p$

(*b*) Blob density measurements were undertaken on relativistic particle tracks in the region $2 \leqslant Z \leqslant 8$, and on the last 20 mm of stopping proton tracks. Blob density

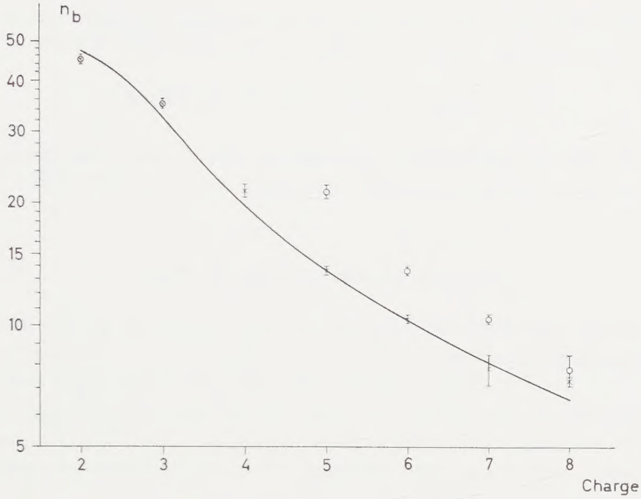

Fig. 2. The relation between the blob density n_b (blobs per 100 microns) and the charge. The crosses represent the results if the charge calibration in Figure 1 is assumed to be correct. The open circles represent the charge dependence assuming that no beryllium nuclei are found in our investigation (see the text). The curve is determined from blob density measurements on stopping proton tracks.

of proton tracks was determined at residual ranges corresponding to Z^2 times minimum ionization and plotted against corresponding Z values in Figure 2. To transform the residual range of protons to ionization, the Emulsion Tables of Barkas and Young [11] were used. In Figure 2 the results of blob density measurements on relativistic multiply-charged particles are also shown. As is seen in the figure, the blob density for these particles falls very close to the proton curve if the charge calibration in Figure 1 is used, whereas the attempted calibration curve causes the blob densities to fall far away from the curve. The results of blob density measurements shown in Figure 2 thus lend additional, powerful support for the assumption that the attempted calibration curve is incorrect.

(*c*) This attempted calibration would lead to a most peculiar charge distribution in the cosmic radiation, yielding a very large amount of F nuclei and rather few O nuclei.

Even if the fact (*c*) cannot be ascribed any decisive value in the argumentation, it is clear from the discussion above that Figure 1 represents the true dependence of δ-ray density on the charge. Consequently, the charge identification assumed in the photometric investigation [3] is correct.

As may be seen from Figure 1 the calibration curve intersects the N_δ-axis at the point $N_\delta = 0.95$. This means that occasional electron tracks and configurations of grains in the background are counted as δ-rays to an amount of 0.95/100 microns. If N_δ is the experimentally found δ-ray density as expressed in the number of δ-rays per 100 micron track length, and n_δ in the same way is the genuine δ-ray density, the relation between them is

$$N_\delta = 0.95 + n_\delta,$$

provided that the spurious δ-ray density is independent of the charge. The relation between N_δ and the charge, as determined by the method of least squares, is then

$$N_\delta = (0.95 \pm 0.15) + (0.139 \pm 0.003)\, Z^2.$$

No correction of the experimentally determined δ-ray densities as to the spurious δ-ray density has been undertaken, the quantity used for the δ-ray density below being throughout N_δ.

Analysis of the sources of error

In order to study the different sources of error involved in the method of δ-ray counting the standard deviations have been computed for all tracks lying in the charge interval $5 \leqslant Z \leqslant 10$. Every cell of 100 counted δ-rays has been treated as an independent determination of δ-ray density. The computed deviations σ are given in Table 2 together with the coefficients of variation, $v = 100\,\sigma/N_\delta$. The errors indicated are standard errors, which are used throughout in the discussions below.

Table 2.

Z	N_δ	σ	v
5	4.57 ± 0.12	0.54 ± 0.09	11.8 ± 2.0
6	6.25 ± 0.14	0.75 ± 0.10	12.0 ± 1.6
7	7.79 ± 0.22	0.95 ± 0.15	12.2 ± 2.0
8	9.83 ± 0.22	1.24 ± 0.16	12.6 ± 1.6
9	11.90 ± 0.23	1.19 ± 0.17	10.0 ± 1.4
10	14.93 ± 0.23	1.59 ± 0.17	10.6 ± 1.1

As may be seen from the table, v is fairly constant in the charge interval $5 \leqslant Z \leqslant 8$, where it has a mean value of $(12.2 \pm 0.9)\,\%$. This value indicates that there must exist some additional source of error besides the statistical fluctuations, which are 10 %. The possible sources of errors are:

(*a*) Spread in the particle energy.

(*b*) Subjective variations in the counting procedure.

(*c*) Fluctuations in the sensitivity between different plates and local irregularities in the emulsions.

The size of factor (*a*) may be estimated by considering the energy of vertically incident particles at the top of the atmosphere. Assuming an integral energy spectrum of the form

$$N(\geqslant \varepsilon) = \text{const.}\ (1 + \varepsilon)^{-1.1},$$

where ε = energy per nucleon in GeV, and using for δ-ray density the formula $n_\delta = \text{const.}\ Z^2/\beta^2$, the expected spread in δ-ray density at $\lambda = 41°$ will be given by the distribution shown in Figure 3. According to this distribution the mean particle velocity is $\beta = 0.97$. The coefficient of variation in δ-ray density is found to be 4.7 %. The use of an exponent of -1.5 instead of -1.1 in the above formula does not change the value of the coefficient of variation. The value of 4.7 % is consistent with

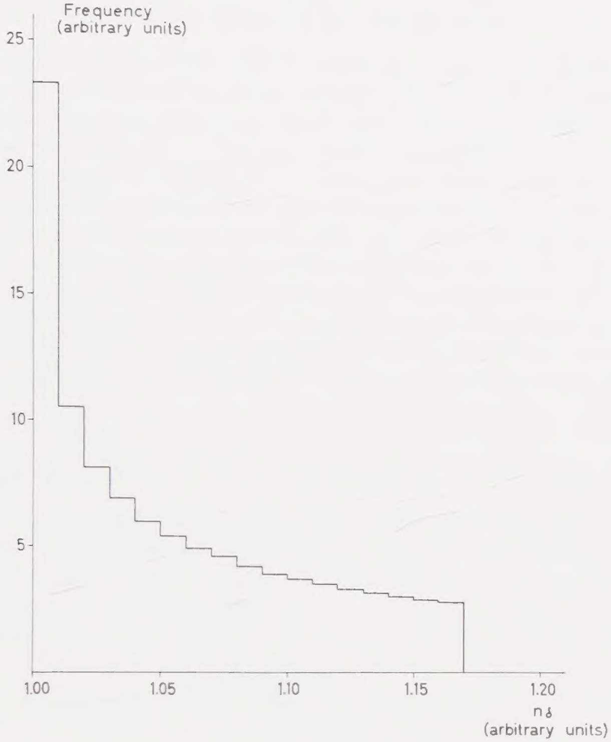

Fig. 3. The expected δ-ray density distribution for tracks of primary particles of a given charge at the top of the atmosphere. The histogram is based on an energy spectrum of the form $N(\geqslant \varepsilon) = \text{const.}\ (1+\varepsilon)^{-1.1}$ with a cut-off at 1.5 GeV per nucleon. The formula $n_\delta = \text{const.}\ Z^2/\beta^2$ is used for the δ-ray density.

the results of Appa Rao *et al.* [12], who took into consideration the exact Mott's formula and the east–west asymmetry as well. The coefficient of variation increases somewhat as the particles penetrate through the atmosphere and the stack. The part of the experimental coefficient of variation which depends on the spread in energy is estimated to be 5.4 % after the passage of 18 g/cm^2 air and 12 g/cm^2 emulsion. Thus the pure statistical variations and the spread in the energy of the particles together result in a coefficient of variation of 11.4 % which is to be compared with the experimental value of (12.2 ± 0.9) %. These figures indicate that it is not impossible that some further sources of error really exist. The remaining part of the coefficient of variation, which must be ascribed to these errors, is 4.5 %.

The source of error (*b*) is discussed in the chapter dealing with reproducibility. From these discussions and from the particular conditions for the counting procedure it is clear that the significance of this factor is greatly minimized. The differences between successive countings on the same portion of a track indicate that the still existing subjective variations amount to about 2 %. The remaining error must be ascribed to the factor (*c*). As it is very difficult to make numerical calculations of the size of this factor, no attempt has been made to estimate the contribution to the total error from this factor.

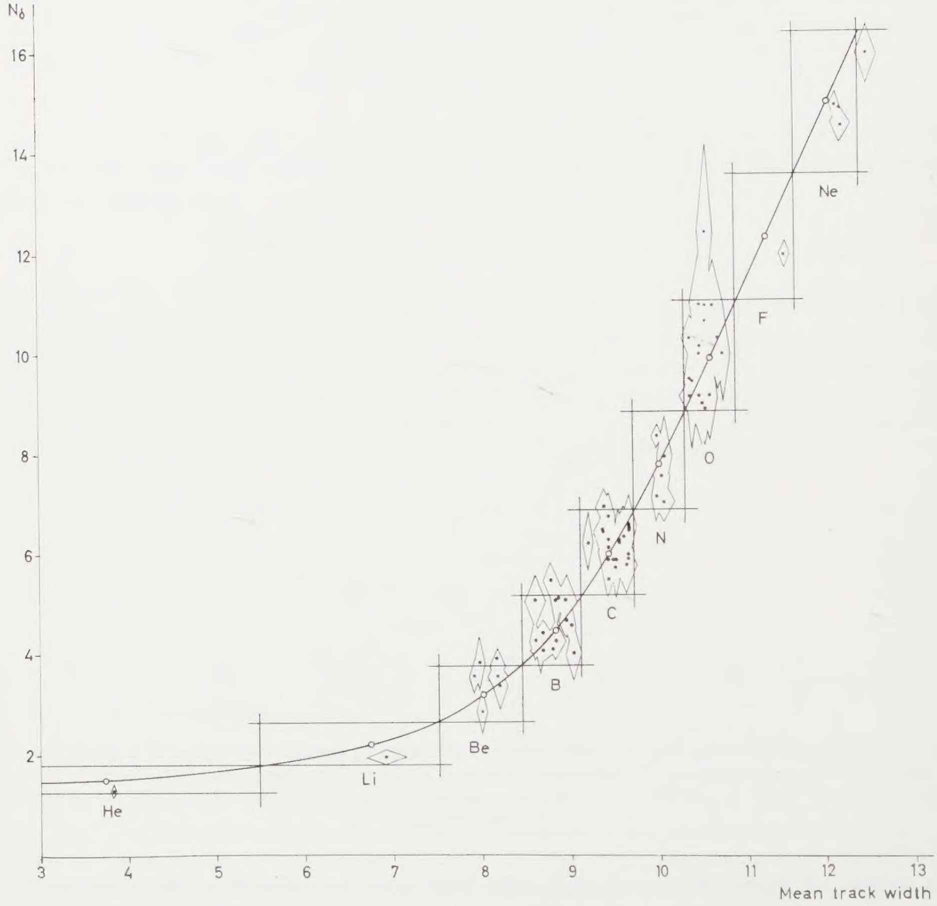

Fig. 4. The relation between δ-ray density and mean track width. Each dot represents measurements on one track. The diamond symbols indicate the errors in both directions. The curve is the locus of points with the same charge according to both methods. The open circles indicate the positions of integral charge values. Lines are inserted on half-integral charge values.

Leaving aside the question of how the compound part of the coefficient of variation in excess of the statistically expected value of 10 % is distributed among the different sources of error, its magnitude is found to be (7.0 ± 1.6) %. Consequently in this experiment the standard deviation in δ-ray counting can never fall below some value around 7 %. In order to reach this value, about 1000 δ-rays should be counted for each track. The measurements would then be drawn out in time and the error introduced by subjective variations in counting procedure would probably increase. Therefore, it seems to be highly plausible that the value of 7 % for the standard deviation represents an upper limit of accuracy.

Comparison with MTW measurements

In Figure 4 the mean track width (MTW) value for tracks used in this investigation is plotted against the corresponding δ-ray density. Each track is represented by one dot in the diagram. The errors for each track are indicated in both directions, thus leading to rhombic fields of errors for each track. In general, 100 δ-rays are counted in each track, but in case of charges $Z \geqslant 7$ often more δ-rays are counted per track in order to secure sufficient statistics for the determination of the charge. The curve drawn in the figure is the locus of points with the same charge according to both methods. Lines are inserted to indicate half-integral values of the charge. As is seen in the figure, the photometric method is throughout superior to the δ-ray method, if only 100 δ-rays are counted per track. A projection of the experimental points on the MTW-axis would result in a charge spectrum with excellent resolution whereas a corresponding projection on the N_δ-axis would lead to a spectrum very difficult to decipher.

The charge spectrum in δ-ray counting and in MTW measurements

The best way of giving an idea of what the charge spectrum from δ-ray counting can be expected to look like is to calculate it by using the frequency of charges as found in the photometric investigation and the standard deviations in the δ-ray density. The calculation has been restricted to the charge interval $4 \leqslant Z \leqslant 10$. The frequency of particles in this interval is given in Table 3 [3]. The standard deviations given in Table 2 are used for charges $5 \leqslant Z \leqslant 8$. For the charges $Z = 4$, 9, and 10 the standard deviations have been computed, assuming that the coefficient of variation is 12.2 %. By means of these standard deviations the normal distribution curves are calculated for each charge. The curves are normalized to the particle frequency given in Table 3 and represented in Figure 5. In this figure the sum of the individual distributions is also given. The locations of the different charges are taken from Figure 1. If the distribution in Figure 5 is accepted, a large amount of elements with $Z = 7$ and $Z = 9$ is found contrary to the results of the photometric measurements [3]. The charge resolution in the region $4 \leqslant Z \leqslant 8$, obtained in the photometric investigation, is shown in Figure 6.

Table 3. Frequency of particles in the charge interval $4 \leqslant Z \leqslant 10$.

Particle	Be	B	C	N	O	F	Ne
Frequency	6	17	27	5	21	1	4

In order to form an idea of the charge spectrum in the most favorable case of δ-ray counting, a curve corresponding to the one represented in Figure 5 has been computed, assuming that the coefficient of variation is 7 % throughout the spectrum. The results are represented in Figure 7.

In spite of the insufficient resolution in the charge spectra computed an attempt was made to estimate the relative frequencies of the different charges in the interval $5 \leqslant Z \leqslant 9$. To each charge Z the region $(Z - 0.5, Z + 0.5)$ was assigned, as indicated by the lines on the charge scale of Figures 5 and 7. The total distribution curves were

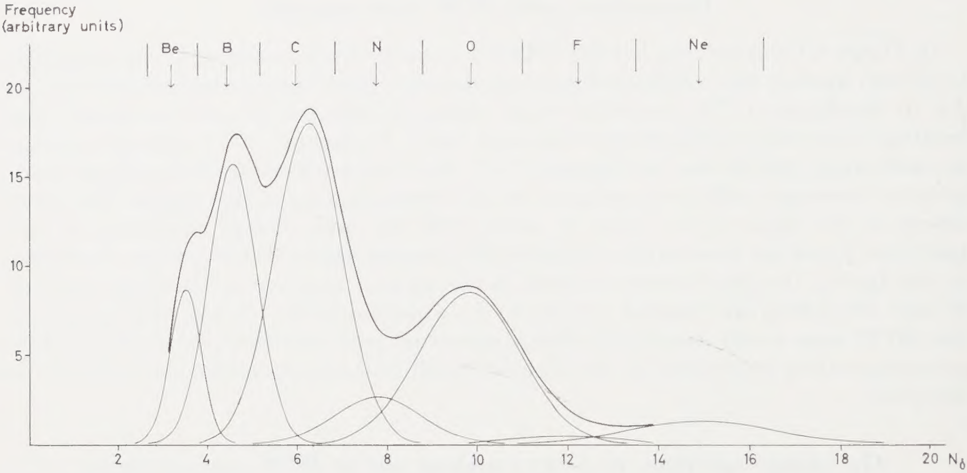

Fig. 5. The individual distribution curves and the total distribution curve (the charge spectrum) in the region $4 \leqslant Z \leqslant 10$ with 100 δ-rays counted per track. The charge scale at the top of the figure with inserted positions of integral and half-integral charge values is taken from Figure 1.

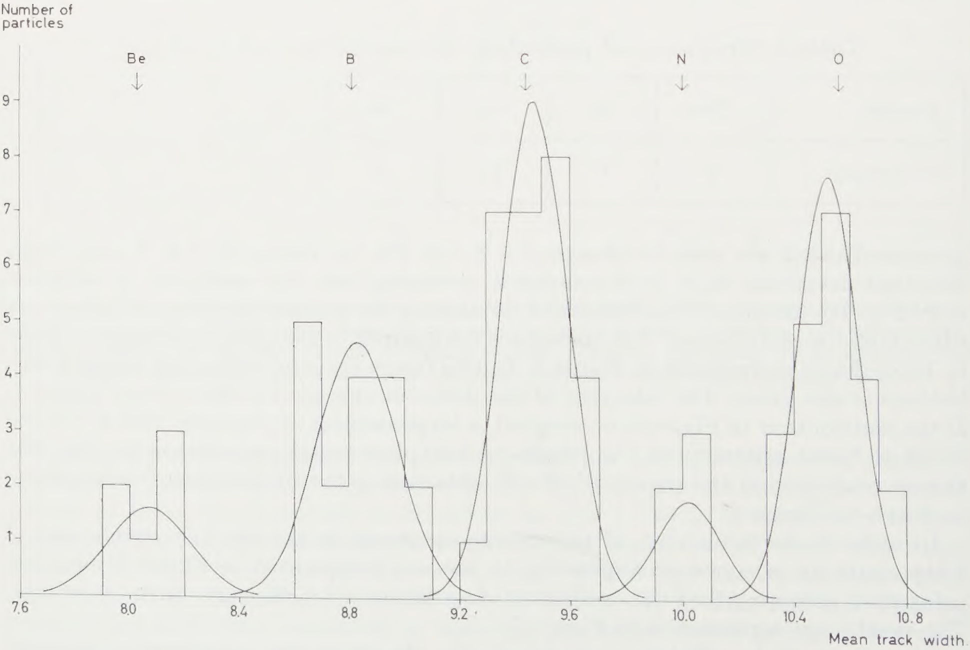

Fig. 6. The photometrically determined charge spectrum in the region $4 \leqslant Z \leqslant 8$ with individual distribution curves. For the charges $Z = 5$, 6, and 8 the standard deviations have been determined experimentally, for $Z = 4$ and $Z = 7$ the standard deviations are computed from data for other groups.

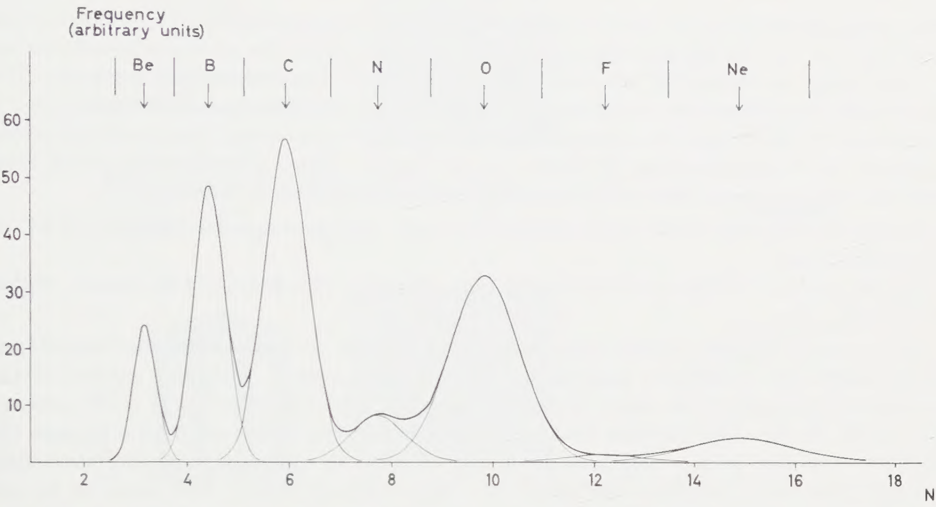

Fig. 7. The expected individual distribution curves and the total distribution curve (the charge spectrum) in the region $4 \leqslant Z \leqslant 10$ calculated with standard deviation $= 7\%$, corresponding to about 1000 δ-rays per track.

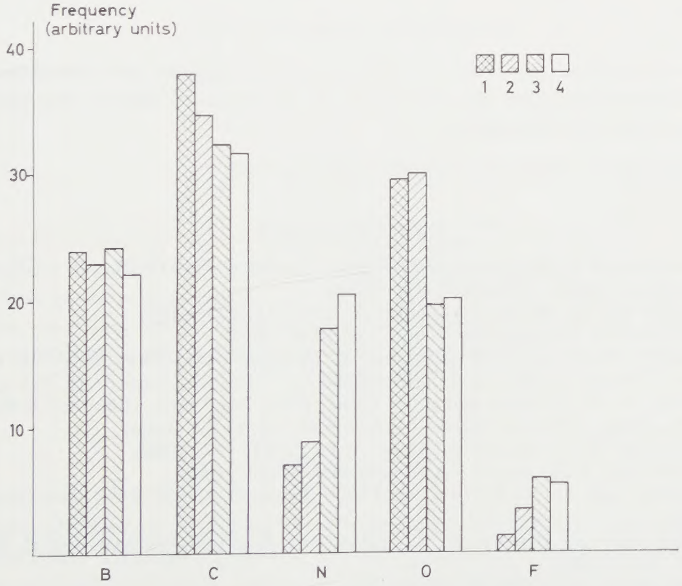

Fig. 8. Comparison of charge spectra in the region $5 \leqslant Z \leqslant 9$, determined by the photometric method and by the δ-ray method.

Column 1: Charge spectrum obtained in the photometric investigation (Table 3).
Column 2: Expected charge spectrum with 1000 δ-rays counted (Figure 7).
Column 3: Charge spectrum with 100 δ-rays counted (Figure 5).
Column 4: The mean of seven charge spectra obtained with the δ-ray method (Bombay [12], Bristol [13, 14], Göttingen [15, 16], Minnesota [17], and Rochester [18]).

The total number of particles in each column is normalized to 100.

integrated between these limits and the results were normalized to 100 particles in the region $5 \leqslant Z \leqslant 9$. In Figure 8 the results are compared with the photometrically determined charge spectrum [3], which is normalized to the same number of particles. The agreement is satisfactory in the case of 1000 δ-rays, but very poor for charges $Z > 6$ in the case of 100 δ-rays. As a comparison the mean of seven other charge distributions obtained by δ-ray counting is shown in the figure. These distributions have been selected in such a way that the following conditions have been realized:

(*a*) the plates must have been exposed as near the geomagnetic latitude of 41° N as possible, and

(*b*) the particle detection must have been entirely effective in the charge region $5 \leqslant Z \leqslant 9$.

It is seen in Figure 8 that the mean of the results of these seven investigations, which fulfil the conditions mentioned [12–18], well agrees with the results of the present investigation. In three of the seven investigations 100 δ-rays were counted per track, in two the number has been some hundreds, whereas in two papers the number of δ-rays is not reported. However, it may be assumed that the statistical accuracy for each track in the mean has been approximately the same as in our investigation. Thus the results shown in Figure 8 are not contradictory to the assumption that the frequency of particles with $Z = 7$ and $Z = 9$ has in reality been very low in the seven investigations as well as in our photometric investigation.

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Department of Physics, University of Lund, February 1960.

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